

# An absolute polarimeter for high energy protons

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## Abstract

A study of the spin asymmetries for polarized elastic proton proton collisions in the electromagnetic hadronic interference (CNI) region of momentum transfer provides a method of self calibration of proton polarization. The method can be extended to non-identical spin half scattering so that, in principle, the polarization of a proton may be obtained through an analysis of its elastic collision with a different polarized particle,  $^3\text{He}$ , for instance. Sufficiently large CNI spin asymmetries provide enough information to facilitate the evaluation of nearly all the helicity amplitudes at small  $t$  as well as the polarization of both initial spin half fermions. Thus it can serve equally well as a polarimeter for  $^3\text{He}$ .

# 1 Introduction

The RHIC collider in its  $pp$  mode is a unique machine. It opens a new frontier in the use of spin for the study of hadronic physics. Using its polarized beams a whole new range of tests of the Standard Model will become feasible, and much new information about the detailed partonic structure of the nucleon will emerge [1]. It will also be possible to answer intriguing questions concerning the relationship between  $pp$  and  $\bar{p}p$  total cross-sections and real parts of forward amplitudes, questions which are relevant to attempts to understand certain aspects of non-perturbative QCD [2]. The entire rich program relies upon an accurate determination of the polarization of the proton beams, hopefully to an accuracy of  $\pm 5\%$ , a matter which is far from trivial.

Much effort has gone into attempts to devise a polarimeter with the required accuracy. The attractiveness of using a Coulomb Nuclear Interference (CNI) polarimeter to measure the polarization at very high energies stems from the belief, as folklore has it, that all hadronic helicity-flip amplitudes are negligible at high energies. In that case, and ignoring the real part of the non-flip amplitude, the analyzing power  $A_N$  has a maximum at  $t = t_p \equiv -8\pi \sqrt{3} \alpha / \sigma_{\text{tot}}$  [3] and its value at the peak is

$$A_N^{\text{max}} = \frac{(-3t_p)^{1/2}}{2m} \cdot \left(\frac{\kappa}{2}\right) \quad (1)$$

where  $\kappa = \mu - 1$  is the anomalous magnetic moment of the proton and  $m$  its mass. (At high energies  $A_N^{\text{max}}$  is between 4% and 5%.) Thus, in the above scenario, one has an almost perfectly calibrated polarimeter. The problem is that if the helicity flip amplitudes are non-negligible then  $\kappa/2$  in Eq. (1) has to be replaced by

$$\frac{\kappa}{2} \rightarrow \frac{\kappa}{2} - I_5 + \frac{\kappa}{2} I_2 \quad (2)$$

where  $I_2$  and  $I_5$  essentially measure the size of the imaginary parts of  $\phi_2$  and  $\phi_5$  relative to the imaginary part of the dominant non-flip amplitude  $\frac{1}{2}(\phi_1 + \phi_3)$ . The precise definitions of  $I_2$  and  $I_5$  are given in terms of helicity amplitudes in Eq. (8) to Eq. (11) below. The  $\phi_i$ 's are defined in [4]. Thus the efficacy of the CNI Polarimeter rests upon being able to demonstrate that  $I_5$  and  $I_2$  are negligible or knowing precise values for them. In a very comprehensive study of this issue [5] we concluded that while it was likely that  $I_2$  can be neglected, it was not possible to exclude the possibility that  $|I_5| \approx 15\%$  implying that the value of  $A_N^{\text{max}}$  can not be calculated to the desired accuracy of  $\pm 5\%$ . (A study of Coulomb interference with polarized protons, from a rather different point of view, has been made by Jakob and Kroll [6].)

However, in the course of this study we discovered the extraordinary fact that  $pp$  elastic scattering is self-calibrating, in the sense that a measurement of a sufficient number of spin-dependent observables at very small  $t$  using a polarized beam and target, but without a priori knowledge of the magnitude of the polarizations, will determine not only the value of most of the helicity amplitudes at small  $t$ , but, remarkably, also the values of the beam and target polarizations. This new approach to polarimetry was discussed briefly in [5], but without adequate analysis of the importance of certain correction terms.

It is likely that some of the helicity amplitudes are very small at RHIC energies, in which case there are small correction terms that were not taken into account in [5] which should be included. It is also possible that much larger spin asymmetries exist in the elastic collision of protons with some other spin 1/2 fermion e.g.  $^3\text{He}$ , so that a more accurate determination of the polarization might be possible via such a reaction. Finally, as compared with [5], we have discovered a more direct way of evaluating the polarization from the measured asymmetries. We therefore present here a detailed analysis of the  $pp$  case using a notation which allows an immediate generalization to the case of non-identical fermions, and we include correction terms which may be important at RHIC energies. We also present a simplified method of obtaining the polarization.

The method we suggest involves the taking of ratios of experimentally determined asymmetries, some of which may be very small. Because so little is known about the helicity amplitudes at high energy we are unable to study this question quantitatively. It will turn out, however, that there are several different ways of determining the polarization of the initial particles and one will have to discover pragmatically which of these provides the most accurate determination. In practice, one would use the values of all the measured asymmetries to extract a best fit to the polarizations.

The method requires the use of asymmetries with both longitudinal and transversely polarized beams. It will not succeed unless data are available with both configurations. Here we work only in order  $\alpha$  and so only amplitudes that are large compared to the next order correction can be determined from the formulas given below. (This could probably be improved upon if necessary: at present experiment will probably not be able to probe amplitudes below that size and so we have not pressed on in this direction.) We assume that the polarized beams have the same degree of polarization in either configuration: since they are produced from the same initial configuration by rotation this is almost certainly true.

Although it is likely that the two beams in a  $pp$  collider have the same polarization, our formulae will allow for different polarizations  $P$  and  $P'$ . This distinction will surely be important for beams of different species.

By now there are other methods that will be available for colliding proton beam polarimetry and which may be simpler and more practical than the one we discuss here. Nevertheless, we present the method because it is elegant and it is interesting that  $P$  can be determined in this way. Furthermore, it may turn out that this is the best way for  $^3\text{He}$  polarimetry, which will very likely be needed eventually at RHIC. Section 5 presents an alternative method for  $^3\text{He}$  polarimetry; it has some model dependence but will almost certainly be practical for high energy  $p\ ^3\text{He}$  collisions.

## 2 Identical Fermions

To see the principles of the method, first consider the elastic collision of identical protons with mass  $m$  and anomalous magnetic moment  $\kappa$ . The usual five helicity

amplitudes  $\Phi_j(s, t)$  [4] are normalized so that

$$\sigma_{\text{tot}}(s) = \frac{4\pi}{\{s(s - 4m^2)\}^{1/2}} \text{Im}(\Phi_1 + \Phi_3)_{t=0} \quad (3)$$

and

$$\frac{d\sigma(s, t)}{dt} = \frac{2\pi}{s(s - 4m^2)} \{|\Phi_1|^2 + |\Phi_2|^2 + |\Phi_3|^2 + |\Phi_4|^2 + 4|\Phi_5|^2\} \quad (4)$$

At the very small values of  $t$  we are interested in, the interference between the strong and electromagnetic forces is crucial, and can be taken into account by writing

$$\Phi_j = \phi_j e^{-i\delta} + \phi_j^{em} \quad (5)$$

where  $\phi_i$  are the hadronic amplitudes and  $\delta$  is the Coulomb phase which was shown in [7] to be the same for all helicity amplitudes. Its role will be discussed in Section 4. In Eq. (5) the  $\phi_i^{em}$  are one photon exchange amplitudes and are real. We are, of course, primarily interested in learning about the hadronic amplitudes  $\phi_i$  for which we shall write expressions valid for very small  $t$ . We define

$$\phi_{\pm} \equiv \frac{1}{2}(\phi_1 \pm \phi_3) \quad (6)$$

so that at high energies  $\text{Im}\phi_+(s, 0)$  is large and positive and should be the dominant amplitude. We shall put

$$\phi_+(s, t) = \text{Im}\phi_+ \left( \rho e^{\bar{B}t/2} + i e^{Bt/2} \right) \quad (7)$$

where  $\text{Im}\phi_+$  is short for  $\text{Im}\phi_+(s, 0)$ ,  $\rho$  is the usual ratio of real to imaginary parts of the forward spin averaged amplitude and  $B$  is the slope of  $d\sigma/dt$ . Here we allow the possibility that the real and imaginary parts have somewhat different slopes. We also introduce scaled amplitudes as follows:

$$\left( R_2 e^{\bar{B}_2 t/2} + i I_2 e^{B_2 t/2} \right) = \frac{\phi_2(s, t)}{2 \text{Im}\phi_+} \quad (8)$$

$$\left( R_- e^{\bar{B}_- t/2} + i I_- e^{B_- t/2} \right) = \frac{\phi_-(s, t)}{\text{Im}\phi_+} \quad (9)$$

$$\left( R_5 e^{\bar{B}_5 t/2} + i I_5 e^{B_5 t/2} \right) = \left( \frac{m}{\sqrt{-t}} \right) \frac{\phi_5(s, t)}{\text{Im}\phi_+} \quad (10)$$

$$\left( R_4 e^{\bar{B}_4 t/2} + i I_4 e^{B_4 t/2} \right) = \left( \frac{m^2}{-t} \right) \frac{\phi_4(s, t)}{\text{Im}\phi_+} \quad (11)$$

Note the factor of 2 in Eq. (8), which is introduced for later convenience. Note too that we have taken out explicit kinematic factors of  $\sqrt{-t}$  and  $t$  from  $\phi_5$  and  $\phi_4$  respectively since for the strong interaction amplitudes  $\phi_5 \propto \sqrt{-t}$  and  $\phi_4 \propto t$  as  $t \rightarrow 0$ . The  $R_j$  and  $I_j$  may vary with  $s$ .

We shall need to consider various experimentally determined asymmetries,  $PA_N \frac{d\sigma}{dt}$ ,  $P'A_N \frac{d\sigma}{dt}$ ,  $PP'A_{NN} \frac{d\sigma}{dt}$ ,  $PP'A_{LL} \frac{d\sigma}{dt}$ ,  $PP'A_{SS} \frac{d\sigma}{dt}$  and  $PP'A_{SL} \frac{d\sigma}{dt}$  [8]. Those contain singular terms as  $t \rightarrow 0$  arising from the interference between one photon exchange and hadronic amplitudes. To order  $\alpha$  the asymmetries  $A_{NN} \frac{d\sigma}{dt}$ ,  $A_{LL} \frac{d\sigma}{dt}$  and  $A_{SS} \frac{d\sigma}{dt}$  are singular as  $1/t$  while  $A_N \frac{d\sigma}{dt}$  and  $A_{SL} \frac{d\sigma}{dt}$  vary as  $1/\sqrt{-t}$ . We shall also require the total cross-section differences  $PP'\Delta\sigma_T$  and  $PP'\Delta\sigma_L$ . Keeping therefore the most singular terms as  $t \rightarrow 0$  and working to order  $\alpha$ , which implies ignoring the Coulomb phase  $\delta$  in (5) (the re-insertion of  $\delta$  will be discussed in Section 3), we may write the following expressions for the various experimental observables:

$$\Delta_T \equiv -\frac{1}{2}PP' \frac{\Delta\sigma_T}{\sigma_{\text{tot}}} = PP'I_2 \quad (12)$$

$$\Delta_L \equiv \frac{1}{2}PP' \frac{\Delta\sigma_L}{\sigma_{\text{tot}}} = PP'I_- \quad (13)$$

For the differential cross section, with slope parameter  $B$  for proton proton collisions, which has an expansion in powers of  $t$  beginning with the photon pole and interference terms we write the form [9]

$$\mathcal{I}_0 \equiv \left( \frac{t}{e^{Bt}\sigma_{\text{tot}}} \right) \frac{d\sigma}{dt} = \frac{4\pi}{\sigma_{\text{tot}}} \frac{\alpha^2}{t} + \alpha a_0 + \frac{\sigma_{\text{tot}}}{8\pi} b_0 t. \quad (14)$$

Here, and in the expressions to follow, it is assumed that coefficients like  $a_j$  and  $b_j$  (see Table 1) are essentially  $t$ -independent in the very small range of  $t$  under consideration. (Note that we have changed the signs of some of the definitions of these quantities from [5] for clarity. Also, there was a sign error in the table entry for  $A_N$  in [5] which has been corrected here.) This key assumption can be tested experimentally by confirming that the observables really do follow the  $t$ -dependence given. The quantities  $b_j^{(0)}$  are calculated assuming that all the hadronic slopes are equal and equal to the electromagnetic slopes as well. This will be corrected for below. For asymmetries with initial protons polarized parallel or anti-parallel along the same axis,  $N$  (normal to the scattering plane) or  $L$  (longitudinal), we have

$$PP'A_{NN}\mathcal{I}_0 = \alpha a_{NN} + \frac{\sigma_{\text{tot}}}{8\pi} b_{NN} t + \dots \quad (15)$$

$$PP'A_{LL}\mathcal{I}_0 = \alpha a_{LL} + \frac{\sigma_{\text{tot}}}{8\pi} b_{LL} t + \dots \quad (16)$$

Spin observables with both initial protons polarized either way along perpendicular axes have a similar power series expansion

$$PP'A_{SL}\mathcal{I}_1 = \alpha a_{SL} + \frac{\sigma_{\text{tot}}}{8\pi} b_{SL} t \quad (17)$$

where again the coefficients  $a_{SL}$ ,  $b_{SL}$  are given in Table 1 and we have defined

$$\mathcal{I}_1 \equiv \frac{m\sqrt{-t}}{e^{Bt}\sigma_{\text{tot}}} \frac{d\sigma}{dt} \quad (18)$$

Table 1: Expressions for the coefficients  $a_j$  and  $b_j^{(0)}$  relevant to the measured observables for  $pp$  elastic scattering

Observable	$a_j$	$b_j^{(0)}$
$\mathcal{I}_0$	$\rho$	$\frac{1}{2}(1 + \rho^2 + R_-^2 + I_-^2) + R_2^2 + I_2^2$
$PP'A_{NN}\mathcal{I}_0$	$PP'R_2$	$PP'[R_2(\rho + R_-) + I_2(1 + I_-)]$
$PP'A_{LL}\mathcal{I}_0$	$PP'R_-$	$PP'[\rho R_- + I_- + R_2^2 + I_2^2]$
$PP'A_{SL}\mathcal{I}_1$	$-PP'\frac{\kappa}{2}(R_2 + R_-)$	$-PP'[R_5(R_2 + R_-) + I_5(I_2 + I_-)]$
$PA_N\mathcal{I}_1$	$P[\frac{\kappa}{2}(1 + I_2) - I_5]$	$P[-I_5(\rho + R_2) + R_5(1 + I_2)]$
$P'A_N\mathcal{I}_1$	$P'[\frac{\kappa}{2}(1 + I_2) - I_5]$	$P'[-I_5(\rho + R_2) + R_5(1 + I_2)]$

Of course for  $pp$  scattering  $A_{LS} = A_{SL}$ . Note that to the accuracy of our approximations  $A_{NN} = A_{SS}$ , so the latter is not discussed separately. However a measurement of  $A_{SS}$  could be used as a check on the adequacy of our approximations. For spin asymmetries with only one of the initial protons polarized we have

$$PA_N\mathcal{I}_1 = \alpha a_N + \frac{\sigma_{\text{tot}}}{8\pi} b_N t + \dots, \quad (19)$$

$$P'A_N\mathcal{I}_1 = \alpha a_N' + \frac{\sigma_{\text{tot}}}{8\pi} b_N' t + \dots. \quad (20)$$

We write the last pair in this rather pedantic way to emphasize the fact that single spin asymmetries may be measured for either beam and, although the analyzing powers are the same, the polarizations  $P$  and  $P'$  may be different. For the comparable situation discussed in the Section 4, the analyzing powers will be different too, in general. The expressions for the measurable coefficients,  $a_j$ ,  $b_j$ , taken from [5], are given in Tables 1 and 2 in terms of the hadronic amplitudes  $R_j$ ,  $I_j$  and the anomalous magnetic moment  $\kappa$ . Each  $b_j$  is written in the form

$$b_j = b_j^{(0)} + \frac{8\pi\alpha}{\sigma_{\text{tot}}} \Delta b_j \quad (21)$$

with the  $b_j^{(0)}$  given in Table 1 and the  $\Delta b_j$  in Table 2. Note that  $b_{SS}^{(0)} = b_{NN}^{(0)}$ , so it is not listed. We have kept only the largest correction term for  $b_N$ . In these expressions we have not included the Coulomb phase  $\delta$ . We shall explain in Section 3 the consequences of including it. Note that  $\phi_4$  only occurs in the correction terms  $\Delta b_j$ . In Table 2 the parameters  $\beta_1$ ,  $\beta_2$  have the following significance. In order to take into account the  $t$ -dependence of the electromagnetic form factors of the proton we parametrize the Dirac and Pauli form factors, for very small  $t$ , in the form

$$F_1(t) = e^{\beta_1 t}, \quad F_2(t) = e^{\beta_2 t} \quad (22)$$

Table 2: Correction terms  $\Delta b_j$  in  $b_j = b_j^0 + \frac{8\pi\alpha}{\sigma_{\text{tot}}}\Delta b_j$

Observable	$\Delta b_j$
$PP'A_{NN}\mathcal{I}_0$	$PP' \left[ \frac{\kappa^2}{4m^2}\rho + R_2(2\beta_1 + \bar{B}_2/2 - B) - \frac{\kappa}{m}R_5 + \frac{R_4}{2m^2} \right]$
$PP'A_{SS}\mathcal{I}_0$	$PP' \left[ \frac{\kappa^2}{4m^2}R_- + R_2(2\beta_1 + \bar{B}_2/2 - B) - \frac{R_4}{2m^2} \right]$
$PP'A_{LL}\mathcal{I}_0$	$PP' \left[ R_-(2\beta_1 + \bar{B}_-/2 - B) + \frac{\kappa^2}{4m^2}R_2 \right]$
$PP'A_{SL}\mathcal{I}_1$	$-PP' \frac{\kappa}{2} \left[ R_2(\beta_1 + \beta_2 + \bar{B}_2/2 - B) + R_-(\beta_1 + \beta_2 + \bar{B}_-/2 - B) - \frac{R_4}{2m^2} \right]$
$PA_N\mathcal{I}_1$	$P \frac{\kappa}{2} (\beta_1 + \beta_2 - B/2)$

where, it turns out that with  $\Lambda^2 = 0.71 \text{ GeV}^{-2}$  [10],

$$\begin{aligned}\beta_1 &= 2/\Lambda^2 - \kappa/(2m)^2 = 2.31 \text{ GeV}^{-2}, \\ \beta_2 &= 2/\Lambda^2 + 1/(2m)^2 = 3.10 \text{ GeV}^{-2}.\end{aligned}\tag{23}$$

Strictly, in order to use our method, one must have information about the slopes  $\bar{B}_2$  and  $\bar{B}_-$ , which seem very hard to come by, or else to neglect these corrections. Since they always appear in the form of a difference from  $B$  times one of the small amplitudes it seems reasonable to neglect them in what follows. (Of course the correction from the difference between the electromagnetic slopes and  $B/2$  is known and that correction can be applied.)

### 3 The Method Applied to Identical Particles

We assume that  $\sigma_{\text{tot}}$ ,  $d\sigma/dt$ ,  $\rho$ ,  $\Delta_L$  and  $\Delta_T$  (see Eq. (12) and Eq. (13)) are measured, and that some of the coefficients in Eq. (15) to Eq. (20), in particular  $a_{LL}$ ,  $b_{LL}$  and  $a_{NN}$  have been determined. these provide values for the product of the unknown polarizations with the real parts of the non-zero forward amplitudes:

$$\rho = a_o \quad PP'R_2 = a_{NN} \quad PP'R_- = a_{LL}\tag{24}$$

Substituting these results into the expression for  $b_{LL}$  in Tables 1 and 2 one has:

$$\begin{aligned}b_{LL} &= PP' \left[ \rho R_- + I_- + R_2^2 + I_2^2 + \frac{2\pi\alpha\kappa^2}{m^2\sigma_{\text{tot}}}R_2 \right] \\ &= \rho a_{LL} + \Delta_L + (a_{NN}^2 + \Delta_T^2)/PP' + \frac{2\pi\alpha\kappa^2}{m^2\sigma_{\text{tot}}}a_{NN}\end{aligned}\tag{25}$$

from which one obtains an expression for the polarization

$$PP' = (a_{NN}^2 + \Delta_T^2) / \left( b_{LL} - \rho a_{LL} - \Delta_L - \frac{2\pi\alpha\kappa^2}{m^2\sigma_{\text{tot}}}a_{NN} \right).\tag{26}$$

There is an important question which we will try to address. How reliable is Eq. (26) if the helicity amplitudes are small? Although we cannot predict the magnitude of the helicity amplitudes at high energies, consideration of quantum number exchange [5] suggests that  $|\phi_-|$  will be the smallest of the above hadronic amplitudes, and we expect  $|R_-|, |I_-| \ll 1\%$  [11]. For the method to work we need  $|\phi_2|$  to be considerably larger, albeit still small. How small? In order to use Eq. (26) at all it is clear that one must have the uncertainties  $\Delta(b_{LL}) \ll b_{LL}$  and max of  $(\Delta(\Delta_L)$  and  $\Delta(\Delta_T)) \ll$  either  $a_{NN}$  or  $\Delta_T$ . If this is false then our whole approach would break down even if  $R_5$  and  $I_5$  are large. This is because the reaction then becomes analogous to spin 1/2-spin 0 scattering, for which we certainly cannot determine the amplitudes without knowing  $P$ . The requirement that a relative error in  $PP'$  less than 10% is obtained in this way is more demanding; for then using Eq. (26) we will need (assuming for illustration that all the needed observables  $a_i, b_i, \Delta_i$  have the same relative error  $\epsilon$  and adding the errors in quadrature)  $\epsilon < 0.1/\sqrt{5}$ . Thus if  $b_{LL}$  turns out to be less than 0.1, the error will need to be less than about 0.005 which may be doable, but much smaller will probably not be accessible.

If the first condition is satisfied, but the second is not, so that  $PP'$  determined via Eq. (26) has a large uncertainty, there is a more reliable though indirect route to determine the polarization, which involves first determining  $I_2$  and  $I_5$  and then using them to calculate the analyzing power  $a_N$  (see Table 1). Since they will provide only a small correction to the dominant term  $\kappa/2$ , it could well be that the errors in their determination are unimportant in  $A_N$ .

To achieve this, we assume that we have a rough determination of  $PP'$  from (26) and use it to provide an improved result. Let us define the combination of observables in (26) as

$$O_1 \equiv \left( b_{LL} - a_{LL} - \Delta_L - \frac{2\pi\alpha\kappa^2 a_{NN}}{m^2\sigma_{\text{tot}}} \right) / (a_{NN}^2 + \Delta_T^2) \quad (27)$$

Then from (24) we have an estimate of the real parts:

$$R_2 = O_1 a_{NN} \quad R_- = O_1 a_{LL} \quad (28)$$

Also from (12) and (13) we have an estimate of the imaginary parts:

$$I_2 = O_1 \Delta_T \quad I_- = O_1 \Delta_L \quad (29)$$

We now proceed to obtain an estimate of  $R_5$  and  $I_5$ . Using  $a_{SL}$  and the uncorrected expression for  $b_{SL}$  from Table 1 yields

$$R_5 + \left( \frac{I_2 + I_-}{R_2 + R_-} \right) I_5 = \frac{\kappa}{2} \frac{b_{SL}}{a_{SL}} \quad (30)$$

which, via (12), (13) and Table 1 becomes

$$R_5 + \left( \frac{\Delta_T + \Delta_L}{a_{NN} + a_{LL}} \right) I_5 = \frac{\kappa}{2} \frac{b_{SL}}{a_{SL}} \quad (31)$$



The asymmetry  $A_N$  provides a second relation between  $R_5$  and  $I_5$ . From Tables 1 and 2 we have

$$\frac{b_N}{a_N} = \frac{I_5(\rho + R_2) - R_5(1 + I_2)}{I_5 - \frac{\kappa}{2}(1 + I_2)} + \frac{8\pi\alpha(\beta_1 + \beta_2 - B/2)}{\sigma_{\text{tot}}} \quad (32)$$

where a small term has been neglected in the denominator of the connection term. This can be rewritten as

$$R_5 + \left( \frac{c_N - \rho - R_2}{1 + I_2} \right) I_5 = \frac{\kappa}{2} c_N \quad (33)$$

where

$$c_N \equiv \frac{b_N}{a_N} - \frac{8\pi\alpha(\beta_1 + \beta_2 - B/2)}{\sigma_{\text{tot}}} \quad (34)$$

and the known last term in (34) may turn out to be negligible. Solving (31) and (35) for  $I_5$  and  $R_5$ , using (29) we obtain

$$I_5 = \frac{\frac{\kappa}{2} \left( \frac{b_{SL}}{a_{SL}} - c_N \right)}{\left( \frac{\Delta_T + \Delta_L}{a_{NN} + a_{LL}} \right) - \left( \frac{c_N - \rho - a_{NN}O_1}{1 + O_1\Delta_T} \right)} \quad (35)$$

and

$$R_5 = \frac{\kappa}{2} \left\{ \frac{b_{SL}}{a_{SL}} - \frac{\frac{b_{SL}}{a_{SL}} - c_N}{1 - \frac{a_{NN} + a_{LL}}{\Delta_T + \Delta_L} \frac{c_N - \rho - a_{NN}O_1}{1 + \Delta_T O_1}} \right\}. \quad (36)$$

Now if it turns out that  $I_5$  is very small, say  $\lesssim 1\%$ , then the neglect of the correction term in the expression for  $b_{SL}$  renders our estimate inaccurate. However  $I_5$  is then so small compared to  $\kappa/2$  in the expression for  $a_N$  that is irrelevant and should be neglected. If, on the other hand it is “large” i.e.  $\approx 10\%$  then the neglected correction term in  $b_{SL}$  is negligible and the estimate for  $I_5$  (and  $R_5$ ) can be used in computing the hadronic corrections to  $A_N$ . As stressed earlier, since  $I_5$  and  $I_2$  are small corrections compared to  $-\kappa/2$  in  $a_N$ , the errors on them, even if relatively large—and they may be, given the large number of measured quantities that go into Eq. (35) and Eq. (36)—should not be important in computing the value of  $a_N$ . Once  $A_N$  is known the polarization can be measured in the normal way.

Up to the present we have ignored the Coulomb phase  $\delta$  which appears in (5). We now consider the effect of including it. It will turn out that  $\delta$  has *no* effect upon the determination of the polarization given in Section 3. Its only role will be in the evaluation of the hadronic amplitudes themselves. It is given by [15, 16, 17]

$$\delta = -\alpha[\ln(|t|(B/2 + 4/\Lambda^2)) + \gamma] \quad (37)$$

where  $B$  is the logarithmic slope of the  $pp$   $d\sigma/dt$  at  $t = 0$ ,  $\Lambda^2 = 0.71 \text{ GeV}^2$  and  $\gamma$  is Euler’s constant  $\gamma = 0.5772$ . Experimentally  $B \approx 13 \text{ GeV}^{-2}$  at high energies and

increases slowly through the RHIC energy range. The phase  $\delta$  is very small except at extremely tiny values of  $|t|$ , so that we can take

$$\phi_j e^{-i\delta} \approx (\text{Re}\phi_j + \delta \text{Im}\phi_j) + i\text{Im}\phi_j \quad (38)$$

Thus to take into account  $\delta$  one must make the replacements

$$R_j \rightarrow R_j + I_j \delta \quad (39)$$

and

$$\rho \rightarrow \rho + \delta \quad (40)$$

throughout Sections 2 and 3. Note that, strictly speaking,  $\delta$  is a function of  $t$ . However it seems hopeless to try to detect the  $\ln|t|$  behavior in Eq. (37), so we suggest that Eq. (39) and Eq. (40) should be used with an effective  $t$ -independent  $\bar{\delta}$ , equal to the mean value of  $\delta(t)$  for the region of  $t$  covered experimentally. It is important to note that the substitutions Eq. (39) and Eq. (40) do *not* affect the result Eq. (26) for  $PP'$ , nor the result for  $A_N$  calculated on the basis of the Table 1 and 2 formula, in which the results Eq. (35), Eq. (36), Eq. (28) and Eq. (24) are used. The phase  $\delta$  only plays a role in the determination of the hadronic amplitudes *per se*; i.e. any amplitude derived via Eq. (24), Eq. (28), Eq. (35) and Eq. (36) must be reinterpreted on the basis of Eq. (39) and Eq. (40).

## 4 Nonidentical spin half fermions

The above analysis generalizes to the case of the collision of non-identical spin half particles, protons colliding with  $^3\text{He}$  in the CNI region, for example. It may be that the self calibration of proton polarization through the use of proton proton CNI collisions is difficult at particular energies because of unsuitably small spin asymmetries. In such cases it is, in principle, possible to evaluate the proton polarization by a study of the non-identical collision process.

Suppose that the second distinct fermion of mass  $m'$  and polarization  $P'$  has charge  $Ze$ , and anomalous magnetic moment  $\kappa'$ , given in units of proton magnetons, as is usual for nuclei. Real and imaginary scaled amplitudes are introduced as before that now involve the centre of mass momentum  $k$  defined by  $4k^2s = [s - (m' - m)^2][s - (m' + m)^2]$ . A distinct helicity flip amplitude for the second fermion

$$(R_6 e^{\bar{B}_6 t/2} + iI_6 e^{B_6 t/2}) = \left( \frac{m}{\sqrt{-t}} \right) \frac{\phi_6(s, t)}{\text{Im}\phi_+} \quad (41)$$

appears in non-identical elastic collisions.

Using only initial state polarization, with one or both beams polarized, one can measure seven spin dependent asymmetries. We follow the notation of [7]; notice that  $\phi_5$  describes the proton spin-flip while  $\phi_6$  describes the  $^3\text{He}$  spin-flip, while  $A_N$  is the proton analyzing power and  $A'_N$  is the  $^3\text{He}$  analyzing power.

$$\frac{d\sigma}{dt} = \frac{\pi}{2k^2s} \left\{ |\phi_1|^2 + |\phi_2|^2 + |\phi_3|^2 + |\phi_4|^2 + 2|\phi_5|^2 + 2|\phi_6|^2 \right\}$$

$$\begin{aligned}
A_N \frac{d\sigma}{dt} &= -\frac{\pi}{k^2 s} \text{Im} [(\phi_1 + \phi_3) \phi_5^* - (\phi_2 - \phi_4) \phi_6^*] \\
A_{NN} \frac{d\sigma}{dt} &= \frac{\pi}{k^2 s} \text{Re} [\phi_1 \phi_2^* - \phi_3 \phi_4^* - 2\phi_5 \phi_6^*] \\
A'_N \frac{d\sigma}{dt} &= \frac{\pi}{k^2 s} \text{Im} [(\phi_1 + \phi_3) \phi_6^* - (\phi_2 - \phi_4) \phi_5^*] \\
A_{SS} \frac{d\sigma}{dt} &= \frac{\pi}{k^2 s} \text{Re} [\phi_1 \phi_2^* + \phi_3 \phi_4^*] \\
A_{SL} \frac{d\sigma}{dt} &= \frac{\pi}{k^2 s} \text{Re} [(\phi_2 + \phi_4) \phi_5^* - (\phi_1 - \phi_3) \phi_6^*] \\
A_{LS} \frac{d\sigma}{dt} &= \frac{\pi}{k^2 s} \text{Re} [(\phi_1 - \phi_3) \phi_5^* - (\phi_2 + \phi_4) \phi_6^*] \\
A_{LL} \frac{d\sigma}{dt} &= \frac{\pi}{2k^2 s} \left\{ |\phi_1|^2 - |\phi_3|^2 + |\phi_2|^2 - |\phi_4|^2 \right\}. \tag{42}
\end{aligned}$$

For scattering of a proton on a spin-1/2 fermion of charge  $Z$  and anomalous magnetic moment  $\kappa'$  the electromagnetic amplitudes are approximately, for large  $s$  and small  $t$ ,

$$\begin{aligned}
\phi_1^{\text{em}} &= \phi_3^{\text{em}} = \frac{\alpha s Z}{t} F_1 F'_1 \\
\phi_2^{\text{em}} &= -\phi_4^{\text{em}} = \frac{\alpha s \kappa \kappa'}{4m^2} F_2 F'_2 \\
\phi_5^{\text{em}} &= -\frac{\alpha s \kappa Z}{2m\sqrt{-t}} F'_1 F_2 \\
\phi_6^{\text{em}} &= \frac{\alpha s \kappa'}{2m\sqrt{-t}} F_1 F'_2 \tag{43}
\end{aligned}$$

where  $\mu_p = \kappa + 1$  is the proton's magnetic moment, and  $m$  its mass. We recall that the proton electromagnetic form factors are denoted  $F_1(t)$  and  $F_2(t)$  while the Dirac and Pauli form factors of  $^3\text{He}$  we will describe by  $F'_1(t)$  and  $F'_2(t)$ . All of the form factors are normalized to 1 at  $t = 0$ . The form factor  $F'_1$  appears multiplied by the charge of  $^3\text{He}$ ,  $Z = 2$ , but  $F'_2$  does not. Rather it appears with the factor  $\kappa'$ ; note that the magnetic moments are given in units of nuclear magnetons, as is normal for nuclei, and so the proton mass  $m$  appears in the denominator of the expression for  $\phi_6^{\text{em}}$  in Eq. (43) and in the numerator of Eq. (41).

$\mathcal{I}_0$  and  $\mathcal{I}_1$  are defined as before, and

$$\mathcal{I}_0 \left( \frac{t}{e^{Bt} \sigma_{\text{tot}}} \right) \frac{d\sigma}{dt} = \frac{4\pi}{\sigma_{\text{tot}}} \frac{Z^2 \alpha^2}{t} + \alpha a_0 + \frac{\sigma_{\text{tot}}}{8\pi} b_0 t. \tag{44}$$

In addition to  $A_N'$ , which is truly independent of  $A_N$  here, one new non-trivial spin observable for the case of distinct fermions involves  $A_{LS}$  in addition to  $A_{SL}$ , with

Table 3: The left column shows the reaction parameter, an asymmetry in all cases excepting the first, the unpolarized cross section. The second and third columns indicate the related expressions for the coefficients  $a_I$  and  $b_I$ .

Observable	$a_I$	$b_I$
$\mathcal{I}_0$	$Z\rho$	$\frac{1}{2} (1 + \rho^2 + R_-^2 + I_-^2) + R_2^2 + I_2^2$
$PP'A_{NN}\mathcal{I}_0$	$PP'ZR_2$	$PP' [R_2(\rho + R_-) + I_2(1 + I_-)]$
$PP'A_{LL}\mathcal{I}_0$	$PP'ZR_-$	$PP'(\rho R_- + I_- + R_2^2 + I_2^2)$
$PP'A_{SL}\mathcal{I}_1$	$-PP'(\frac{1}{2}\kappa ZR_2 + \frac{1}{2}\kappa'R_-)$	$PP'(-R_2R_5 - I_2I_5 + R_-R_6 + I_-I_6)$
$PA_N\mathcal{I}_1$	$P(\frac{1}{2}\kappa Z + \frac{1}{2}\kappa'I_2 - ZI_5)$	$P(-\rho I_5 + R_5 + R_2I_6 - I_2R_6)$
$PP'A_{LS}\mathcal{I}_1$	$-PP'(\frac{1}{2}\kappa'R_2 + \frac{1}{2}\kappa ZR_-)$	$PP'(R_2R_6 + I_2I_6 - R_-R_5 - I_-I_5)$
$P'A'_N\mathcal{I}_1$	$P'(\frac{1}{2}\kappa' + \frac{1}{2}\kappa ZI_2 + ZI_6)$	$P'(\rho I_6 - R_6 - R_2I_5 + I_2R_5)$

an additional equations corresponding to Eq. (17)

$$PP'A_{LS}\mathcal{I}_1 = \alpha a_{LS} + \frac{\sigma_{\text{tot}}}{8\pi} b_{LS} t \quad (45)$$

The analysis proceeds as in the identical particle case up to the point just before consideration of the evaluation of  $R_5$ . To determine values of  $R_5$  and  $R_6$ , now, we observe from the expressions for  $A_{SL}$  and  $A_{LS}$  in Table 3 that

$$\begin{aligned} a_{SL} \pm a_{LS} &= -PP'\frac{1}{2}(\kappa Z \pm \kappa')(R_2 \pm R_-) \\ b_{SL} \pm b_{LS} &= -PP'(R_5 \mp R_6)(R_2 \pm R_-) - PP'(I_5 \mp I_6)(I_2 \pm I_-). \end{aligned} \quad (46)$$

Upon division, we are led to two linear relations among the four quantities  $R_5$ ,  $R_6$ ,  $I_5$ , and  $I_6$  so that from Eq. (46)

$$(R_5 \mp R_6) + (I_5 \mp I_6) \frac{I_2 \pm I_-}{R_2 \pm R_-} = \frac{1}{2}(\kappa Z \pm \kappa') \frac{b_{SL} \pm b_{LS}}{a_{SL} \pm a_{LS}}. \quad (47)$$

Two further linear relations among  $R_5$ ,  $R_6$ ,  $I_5$ , and  $I_6$  may be obtained from consideration of  $A_N$  and  $A'_N$ .

$$R_5 - I_2R_6 - \rho I_5 + R_2I_6 = \frac{1}{2}(\kappa Z + \kappa'I_2 - 2ZI_5) \frac{b_N}{a_N}, \quad (48)$$

$$I_2 R_5 - R_6 - R_2 I_5 + \rho I_6 = \frac{1}{2} (\kappa' + \kappa Z I_2 + 2 Z I_6) \frac{b'_N}{a'_N}. \quad (49)$$

From the four linear relations Eq. (47–49) values of  $R_5$ ,  $R_6$ ,  $I_5$ , and  $I_6$  may be found in general, just as in Eq. (35) and Eq. (36). A glance at  $PA_N$  and  $P'A'_N$  in Table 3 reveals that the equations

$$a_N = P \left( \frac{1}{2} \kappa Z + \frac{1}{2} \kappa' I_2 - Z I_5 \right) \quad (50)$$

$$a_N' = P' \left( \frac{1}{2} \kappa' + \frac{1}{2} \kappa Z I_2 - Z I_6 \right) \quad (51)$$

provide estimates of the individual polarizations,  $P$  and  $P'$ , of the two distinct fermions participating in the elastic collision. All the ingredients for successful polarimetry are to hand. If the various coefficients  $a_I$  and  $b_I$  involved in the evaluation of the scaled amplitudes are sufficiently non-zero within error, then Eq. (50) indicates the extent of the asymmetry maximum to be expected for polarized protons. The corrections and error discussion go through just as in Section 3.

## 5 An alternative method for $^3\text{He}$ polarimetry

Although the method described here is elegant and totally model-independent, it may fail if the key asymmetries turn out to be too small to measure precisely. In this section, we present an alternative method which depends only on measuring  $A_N$  and  $A'_N$ , asymmetries which will almost certainly be large enough to measure accurately. The price to pay will be that some model dependence will enter, but this dependence can probably be controlled and corrected for [12, 13, 14].

To a first approximation, the amplitude for flipping the spin of  $^3\text{He}$  is the same as that for flipping the neutron spin in  $pn$  elastic scattering. So we begin with the simple parametrization of the  $pn$  amplitudes that corresponds to that used previously for the  $pp$  amplitudes [5].

$$\begin{aligned} \phi_+(s, t) &= \frac{s}{8\pi} \sigma_{\text{tot}}^{pn} (i + \rho^{pn}) e^{B^{pn} t/2}, \\ \phi_5(s, t) &= \frac{\tau_p \sqrt{-t}}{2m} \phi_+(s, t), \\ \phi_6(s, t) &= -\frac{\tau_n \sqrt{-t}}{2m} \phi_+(s, t). \end{aligned} \quad (52)$$

We will neglect  $\phi_-$ ,  $\phi_2$  and  $\phi_4$ . See [5] for a discussion of this approximation. In general,  $\tau_p$  and  $\tau_n$  are complex, but Regge arguments favor nearly real values for high energy. (There is a factor of 2 different in the convention for  $\tau$  from [5]. This is chosen to parallel more closely the anomalous magnetic moment in the electromagnetic amplitudes.)

We will now make several approximations in order to keep the discussion simple. They can be corrected for if a more precise calculation is needed to interpret experiment. First, we assume that we are at sufficiently high energy that the scattering

is given by pure  $I = 0$  exchange. This is almost certainly true at RHIC but corrections can be made when using the approach at lower energy [18]. Thereby we will have  $\tau_p = \tau_n \equiv \tau$  and the other parameters  $\sigma_{\text{tot}}^{pn}, \rho^{pn}, B^{pn}$  will have the values of the corresponding  $pp$  parameters. Next, we will assume independent particle harmonic oscillator wave functions for the nucleons in  $^3\text{He}$  with the two protons coupled to spin singlet so all the spin is carried by the neutron. Finally, we will use the impulse approximation for calculating the  $p^3\text{He}$  scattering in terms of  $pn$  and  $pp$  scattering. These aspects can easily be improved upon by using better wave functions and multiple scattering theory [14]. The results here should not be misleading at the very small  $t$ -values of the CNI peak but the corrections should be calculated if data at larger values of  $|t| \geq 0.1 \text{ GeV}^2$  are used in fitting data to the calculated curve.

Thus we obtain for the hadronic part of the  $p^3\text{He}$  amplitudes

$$\begin{aligned}\phi_+^{p^3\text{He}}(3s, t) &= \frac{9s}{8\pi} \sigma_{\text{tot}}(s) (i + \rho) F_H(t), \\ \phi_5^{p^3\text{He}}(3s, t) &= \frac{\tau\sqrt{-t}}{2m} \phi_+^{p^3\text{He}}(3s, t), \\ \phi_6^{p^3\text{He}}(3s, t) &= -\frac{1}{3} \frac{\tau\sqrt{-t}}{2m} \phi_+^{p^3\text{He}}(3s, t).\end{aligned}\tag{53}$$

Note here that  $s$  denotes the  $s$ -value for the  $pn$  or  $pp$  subsystem and  $\sigma_{\text{tot}}$  denotes the  $pp$  total cross-section, so  $\sigma_{\text{tot}}^{p^3\text{He}}(3s) = (8\pi/3s) \text{Im } \phi_+(3s, 0)$ . The factor 3 in the first equation (and the corresponding 1/3 in the last) results from there being three nucleons in  $^3\text{He}$  (but only one neutron).  $F_H(t) = \exp\{t(B/2 + a^2/4)\}$  where  $a^2$  is the harmonic oscillator parameter, approximately equal to  $57.4 \text{ GeV}^{-2}$ .

In order to calculate the CNI effect, we need also the electromagnetic amplitudes for  $p^3\text{He}$  scattering. These are (we suppress an index referring the  $p^3\text{He}$  for typographical clarity)

$$\begin{aligned}\phi_+^{em}(3s, t) &= \frac{6s\alpha}{t} F_{em}(t), \\ \phi_5^{em}(3s, t) &= -\frac{6s\alpha}{2m\sqrt{-t}} \kappa_p F_{em}(t), \\ \phi_6^{em}(3s, t) &= \frac{3s\alpha}{2m\sqrt{-t}} \kappa_n F_{em}(t),\end{aligned}\tag{54}$$

where  $F_{em}(t) = \exp(a^2 t/4)$ . Note that  $m$  in all these formulas denotes the proton mass. In the second of these equations one might want to use the magnetic moment of  $^3\text{He}$  rather than  $\kappa_n$ ; this is about 10% larger in magnitude. We leave the expressions this way for consistency with our simple approach.

Using these relations one can calculate the quantities  $a_N, a_N', b_N, b_N'$  as well as  $a_0$  and  $b_0$ . Normalizing to the total cross section  $A \sigma_{\text{tot}}$  where  $A$  is the nuclear number

we get

$$a_0 = Z\rho, \quad (55)$$

$$b_0 = \frac{1}{2}A(1 + \rho^2), \quad (56)$$

$$a_N = \frac{1}{2}PZ[\kappa_p - \text{Re}(\tau) - \rho\text{Im}(\tau)], \quad (57)$$

$$b_N = \frac{1}{2}PA\text{Im}(\tau)(1 + \rho^2), \quad (58)$$

$$a_N' = \frac{1}{2}P'\left(\kappa_n - \frac{Z}{A}[\text{Re}(\tau) + \rho\text{Im}(\tau)]\right), \quad (59)$$

$$b_N' = \frac{1}{2}P'\text{Im}(\tau)(1 + \rho^2). \quad (60)$$

Here, of course,  $A = 3$  and  $Z = 2$ ; we write the expressions in this way so that the source of the factors which differ from the  $pp$  case is transparent.

The important result that  $b_N$  and  $b_{N'}$  are simply related to each other results from the assumption of  $I = 0$  dominance of both the flip and non-flip amplitudes. If one knew the proton polarization  $P$  from an independent polarimeter, then this simple relation immediately gives  $P'$ . It is more in the spirit of the present work to use this one experiment to determine everything. Thus

$$\frac{b_N}{b_{N'}} = A \frac{P}{P'} \equiv \mathcal{M}_2. \quad (61)$$

This is to be used then with

$$\frac{a_N}{a_{N'}} = \frac{Z[\kappa_p - \text{Re}(\tau) - \rho\text{Im}(\tau)]}{\kappa_n - \frac{Z}{A}[\text{Re}(\tau) + \rho\text{Im}(\tau)]} \frac{P}{P'} \equiv \mathcal{M}_1 \quad (62)$$

to solve for  $\text{Re}(\tau[1 - i\rho])$  in terms of these two measurable ratios  $\mathcal{M}_1$  and  $\mathcal{M}_2$ :

$$\frac{Z}{A}\text{Re}(\tau[1 - i\rho]) = \frac{\kappa_n\mathcal{M}_1 - \frac{Z}{A}\kappa_p\mathcal{M}_2}{\mathcal{M}_1 - \mathcal{M}_2}. \quad (63)$$

With this result in hand, one can use  $a_N$  to obtain  $P$  and  $a_{N'}$  to obtain  $P'$ . The only fly in the ointment—and it is a pretty serious fly—is that if  $\text{Im}(\tau)$  should vanish the method fails at step one because both  $b_N$  and  $b_{N'}$  then vanish. Regge theory suggests that this is likely to happen at sufficiently high energy, but at RHIC there may remain a sufficient phase difference between the flip and non-flip amplitudes that both  $b_N$  and  $b_{N'}$  are measurably finite. If  $\text{Im}(\tau)$  does vanish, then this method can be used as a polarimeter for  $^3\text{He}$  in conjunction with an independent measurement of the proton beam polarization  $P$ : for then one can use Eq. (57) to obtain  $\tau$  and use it in Eq. (59) to obtain  $P'$ .

## 6 Conclusions

This paper presents a model independent method for determining the polarization of colliding spin 1/2 particles, like or unlike, based on measuring the various single- and

double-spin asymmetries using both longitudinal and transversely polarized beams. No independent information regarding the polarization is needed. The method is dependent on precise measurements of several probably very small quantities, and this may limit its applicability, but the existence of the method is interesting in itself. Finally, an alternative method is given, which depends on some nuclear physics and some high energy approximations, but which should almost certainly be practical for unlike particles.

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